２つの辺の交点

視点を$P(x_0 \quad y_0 \quad z_0)$とする。２つの単壁をとり、一つの単壁のある辺を$A(x_1 \quad y_1 \quad z_1), B(x_2 \quad y_2 \quad z_2)$, 他の単壁の他の辺を$C(x_3 \quad y_3 \quad z_3), D(x_4 \quad y_4 \quad z_4)$とするとき、点P　から見える直線AB　と直線CD　の見掛け上の交点を直線ABと直線CD上に求める。直線CD上の交点の座標を$u(x \quad y \quad z)$と表わし、直線AB 上の交点の座標を$u'(x' \quad y' \quad z')$と表わす。

$P(x_0 \quad y_0 \quad z_0), A(x_1 \quad y_1 \quad z_1)\ B(x_2 \quad y_2 \quad z_2), C(x_3 \quad y_3 \quad z_3), D(x_4 \quad y_4 \quad z_4)$の位置ベクトルをそれぞれ$u_0(x_0 \quad y_0 \quad z_0)$, $u_1(x_1 \quad y_1 \quad z_1), u_2(x_2 \quad y_2 \quad z_2)$, $u_3(x_3 \quad y_3 \quad z_3), u_4(x_4 \quad y_4 \quad z_4)$と表わす。

$u(x \quad y \quad z)$,は点P, A, Bを通る平面上にあるから

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = 0$$

を満たす。従って

$$\begin{vmatrix} x & y & z \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} \qquad ①$$

が成り立つ。

また$u(x \quad y \quad z)$,は直線 C,D を通る直線上にあるから

$$\begin{pmatrix} x-x_3 \\ y-y_3 \\ z-z_3 \end{pmatrix} = k\begin{pmatrix} x_4-x_3 \\ y_4-y_3 \\ z_4-z_3 \end{pmatrix} \qquad ②$$

を満たす。従って

$$x-x_3 = k(x_4-x_3), y-y_3 = k(y_4-y_3), z-z_3 = k(z_4-z_3)$$

これらを①の左辺に代入すると

$$\begin{vmatrix} k(x_4-x_3)+x_3 & k(y_4-y_3)+y_3 & k(z_4-z_3)+z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}$$

ゆえに

$$\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} k = -\begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} \qquad ③$$

が成り立つ。

②と③から k を消去するために②の両辺に$\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}$を掛けると

$$\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x-x_3 \\ y-y_3 \\ z-z_3 \end{pmatrix} = \begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix} k \begin{pmatrix} x_4-x_3 \\ y_4-y_3 \\ z_4-z_3 \end{pmatrix}$$

②を代入して

$$\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x-x_3 \\ y-y_3 \\ z-z_3 \end{pmatrix} = -\begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_4-x_3 \\ y_4-y_3 \\ z_4-z_3 \end{pmatrix}$$

の k を消去した式を得る。

$$\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} + \begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} - \begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix}$$

従って

$$\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{vmatrix} x_4-x_0 & y_4-y_0 & z_4-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} - \begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix}$$

これから$u = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$を求めると

$$\therefore u = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{\begin{vmatrix} x_4-x_0 & y_4-y_0 & z_4-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} - \begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}\begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix}}{\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}}$$

同様に$u'(x' \quad y' \quad z')$,は点P, C, Dを通る平面上にあるから

$$\begin{vmatrix} x'-x_0 & y'-y_0 & z'-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} = 0$$

を満たす。従って

$$\begin{vmatrix} x' & y' & z' \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} \qquad ④$$

が成り立つ。

また$u'(x' \quad y' \quad z')$,は直線 A,B を通る直線上にあるから

$$\begin{pmatrix} x'-x_1 \\ y'-y_1 \\ z'-z_1 \end{pmatrix} = k\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \\ z_2-z_1 \end{pmatrix} \qquad ⑤$$

を満たす。従って

$x'-x_1=k(x_2-x_1), y'-y_1=k(y_2-y_1), z'-z_1=k(z_2-z_1)$

これらを④の左辺に代入すると

$$\begin{vmatrix} k(x_2-x_1)+x_1 & k(y_2-y_1)+y_1 & k(z_2-z_1)+z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} = \begin{vmatrix} x_0 & y_0 & z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}$$

ゆえに

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} k = -\begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} \qquad ⑥$$

が成り立つ。

⑤と⑥から k を消去するために⑤の両辺に$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}$を掛けると

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x'-x_1 \\ y'-y_1 \\ z'-z_1 \end{pmatrix} = \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix} k\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \\ z_2-z_1 \end{pmatrix}$$

⑥を代入して

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x'-x_1 \\ y'-y_1 \\ z'-z_1 \end{pmatrix} = -\begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x_2-x_1 \\ y_2-y_1 \\ z_2-z_1 \end{pmatrix}$$

の k を消去した式を得る。

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

従って

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{vmatrix} x_2-x_0 & y_2-y_0 & z_2-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

これから$u' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$を求めると

$$\therefore u' = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \frac{\begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix}}$$

３点u_0、u、u'　が直線状に並んでいることを示そう。

$$u - u_0 = \frac{\begin{vmatrix} x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} - \begin{vmatrix} x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix}}{\begin{vmatrix} x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix}} - \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

$$= \frac{\begin{vmatrix} x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} - \begin{vmatrix} x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix} - \begin{vmatrix} x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}{\begin{vmatrix} x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} - \begin{vmatrix} x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_4 \\ y_4 \\ z_4 \end{pmatrix} - \begin{vmatrix} x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{vmatrix} x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}{\begin{vmatrix} x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_3 - x_0 \\ y_3 - y_0 \\ z_3 - z_0 \end{pmatrix} - \begin{vmatrix} x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} \begin{pmatrix} x_4 - x_0 \\ y_4 - y_0 \\ z_4 - z_0 \end{pmatrix}}{\begin{vmatrix} x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix}}$$

同様に

$$u' - u_0 = \frac{\begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix}} - \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

$$= \frac{\begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} - \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} - \begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_1 - x_0 \\ y_1 - y_0 \\ z_1 - z_0 \end{pmatrix} - \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} \begin{pmatrix} x_2 - x_0 \\ y_2 - y_0 \\ z_2 - z_0 \end{pmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix}}$$

$X_i = x_i - x_0, Y_i = y_i - y_0, Z_i = z_i - z_1$ (i=1,2,3,4)とおくと

$$u - u_0 = \frac{\begin{vmatrix} X_4 & Y_4 & Z_4 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} - \begin{vmatrix} X_3 & Y_3 & Z_3 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \begin{pmatrix} X_4 \\ Y_4 \\ Z_4 \end{pmatrix}}{\begin{vmatrix} X_4 - X_3 & Y_4 - Y_3 & Z_4 - Z_3 \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix}}$$

$$u' - u_0 = \frac{\begin{vmatrix} X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} - \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix}}{\begin{vmatrix} X_2 - X_1 & Y_2 - Y_1 & Z_2 - Z_1 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix}}$$

$$
u-u_0\text{の分子}=\begin{vmatrix}X_4 & Y_4 & Z_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\begin{pmatrix}X_3\\ Y_3\\ Z_3\end{pmatrix}-\begin{vmatrix}X_3 & Y_3 & Z_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\begin{pmatrix}X_4\\ Y_4\\ Z_4\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}X_4 & Y_4 & Z_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}X_3\\ \begin{vmatrix}X_4 & Y_4 & Z_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}Y_3\\ \begin{vmatrix}X_4 & Y_4 & Z_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}Z_3\end{pmatrix}-\begin{pmatrix}\begin{vmatrix}X_3 & Y_3 & Z_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}X_4\\ \begin{vmatrix}X_3 & Y_3 & Z_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}Y_4\\ \begin{vmatrix}X_3 & Y_3 & Z_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}Z_4\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}X_4X_3 & Y_4X_3 & Z_4X_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_4Y_3 & Y_4Y_3 & Z_4Y_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_4Z_3 & Y_4Z_3 & Z_4Z_3\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\end{pmatrix}-\begin{pmatrix}\begin{vmatrix}X_3X_4 & Y_3X_4 & Z_3X_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_3Y_4 & Y_3Y_4 & Z_3Y_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_3Z_4 & Y_3Z_4 & Z_3Z_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\end{pmatrix}
$$

$$
=\begin{pmatrix}\begin{vmatrix}X_4X_3-X_3X_4 & Y_4X_3-Y_3X_4 & Z_4X_3-Z_3X_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_4Y_3-X_3Y_4 & Y_4Y_3-Y_3Y_4 & Z_4Y_3-Z_3Y_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_4Z_3-X_3Z_4 & Y_4Z_3-Y_3Z_4 & Z_4Z_3-Z_3Z_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}0 & Y_4X_3-Y_3X_4 & Z_4X_3-Z_3X_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_4Y_3-X_3Y_4 & 0 & Z_4Y_3-Z_3Y_4\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\\ \begin{vmatrix}X_4Z_3-X_3Z_4 & Y_4Z_3-Y_3Z_4 & 0\\ X_1 & Y_1 & Z_1\\ X_2 & Y_2 & Z_2\end{vmatrix}\end{pmatrix}=\begin{pmatrix}-(Y_4X_3-Y_3X_4)\begin{vmatrix}X_1 & Z_1\\ X_2 & Z_2\end{vmatrix}+(Z_4X_3-Z_3X_4)\begin{vmatrix}X_1 & Y_1\\ X_2 & Y_2\end{vmatrix}\\ (X_4Y_3-X_3Y_4)\begin{vmatrix}Y_1 & Z_1\\ Y_2 & Z_2\end{vmatrix}+(Z_4Y_3-Z_3Y_4)\begin{vmatrix}X_1 & Y_1\\ X_2 & Y_2\end{vmatrix}\\ (X_4Z_3-X_3Z_4)\begin{vmatrix}Y_1 & Z_1\\ Y_2 & Z_2\end{vmatrix}-(Y_4Z_3-Y_3Z_4)\begin{vmatrix}X_1 & Z_1\\ X_2 & Z_2\end{vmatrix}\end{pmatrix}
$$

$$
=\begin{pmatrix}-\begin{vmatrix}X_1 & Z_1\\ X_2 & Z_2\end{vmatrix}\begin{vmatrix}X_3 & Y_3\\ X_4 & Y_4\end{vmatrix}+\begin{vmatrix}X_1 & Y_1\\ X_2 & Y_2\end{vmatrix}\begin{vmatrix}X_3 & Z_3\\ X_4 & Z_4\end{vmatrix}\\ -\begin{vmatrix}Y_1 & Z_1\\ Y_2 & Z_2\end{vmatrix}\begin{vmatrix}X_3 & Y_3\\ X_4 & Y_4\end{vmatrix}+\begin{vmatrix}X_1 & Y_1\\ X_2 & Y_2\end{vmatrix}\begin{vmatrix}Y_3 & Z_3\\ Y_4 & Z_4\end{vmatrix}\\ -\begin{vmatrix}Y_1 & Z_1\\ Y_2 & Z_2\end{vmatrix}\begin{vmatrix}X_3 & Z_3\\ X_4 & Z_4\end{vmatrix}+\begin{vmatrix}X_1 & Z_1\\ X_2 & Z_2\end{vmatrix}\begin{vmatrix}Y_3 & Z_3\\ Y_4 & Z_4\end{vmatrix}\end{pmatrix}
$$

$$
u'-u_0\text{の分子}=\begin{vmatrix}X_2 & Y_2 & Z_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\begin{pmatrix}X_1\\ Y_1\\ Z_1\end{pmatrix}-\begin{vmatrix}X_1 & Y_1 & Z_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\begin{pmatrix}X_2\\ Y_2\\ Z_2\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}X_2 & Y_2 & Z_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}X_1\\ \begin{vmatrix}X_2 & Y_2 & Z_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}Y_1\\ \begin{vmatrix}X_2 & Y_2 & Z_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}Z_1\end{pmatrix}-\begin{pmatrix}\begin{vmatrix}X_1 & Y_1 & Z_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}X_2\\ \begin{vmatrix}X_1 & Y_1 & Z_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}Y_2\\ \begin{vmatrix}X_1 & Y_1 & Z_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}Z_2\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}X_2X_1 & Y_2X_1 & Z_2X_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\\ \begin{vmatrix}X_2Y_1 & Y_2Y_1 & Z_2Y_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\\ \begin{vmatrix}X_2Z_1 & Y_2Z_1 & Z_2Z_1\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\end{pmatrix}-\begin{pmatrix}\begin{vmatrix}X_1X_2 & Y_1X_2 & Z_1X_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\\ \begin{vmatrix}X_1Y_2 & Y_1Y_2 & Z_1Y_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\\ \begin{vmatrix}X_1Z_2 & Y_1Z_2 & Z_1Z_2\\ X_3 & Y_3 & Z_3\\ X_4 & Y_4 & Z_4\end{vmatrix}\end{pmatrix}
$$

$$
=\begin{pmatrix}
\begin{vmatrix} X_2X_1-X_1X_2 & Y_2X_1-Y_1X_2 & Z_2X_1-Z_1X_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix} \\
\begin{vmatrix} X_2Y_1-X_1Y_2 & Y_2Y_1-Y_1Y_2 & Z_2Y_1-Z_1Y_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix} \\
\begin{vmatrix} X_2Z_1-X_1Z_2 & Y_2Z_1-Y_1Z_2 & Z_2Z_1-Z_1Z_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix}
\end{pmatrix}
=\begin{pmatrix}
\begin{vmatrix} 0 & Y_2X_1-Y_1X_2 & Z_2X_1-Z_1X_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix} \\
\begin{vmatrix} X_2Y_1-X_1Y_2 & 0 & Z_2Y_1-Z_1Y_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix} \\
\begin{vmatrix} X_2Z_1-X_1Z_2 & Y_2Z_1-Y_1Z_2 & 0 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \end{vmatrix}
\end{pmatrix}
=\begin{pmatrix}
-(Y_2X_1-Y_1X_2)\begin{vmatrix} X_3 & Z_3 \\ X_4 & Z_4 \end{vmatrix}+(Z_2X_1-Z_1X_2)\begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix} \\
(X_2Y_1-X_1Y_2)\begin{vmatrix} Y_3 & Z_3 \\ Y_4 & Z_4 \end{vmatrix}+(Z_2Y_1-Z_1Y_2)\begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix} \\
(X_2Z_1-X_1Z_2)\begin{vmatrix} Y_3 & Z_3 \\ Y_4 & Z_4 \end{vmatrix}-(Y_2Z_1-Y_1Z_2)\begin{vmatrix} X_3 & Z_3 \\ X_4 & Z_4 \end{vmatrix}
\end{pmatrix}
$$

$$
=\begin{pmatrix}
-\begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}\begin{vmatrix} X_3 & Z_3 \\ X_4 & Z_4 \end{vmatrix}+\begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}\begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix} \\
-\begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}\begin{vmatrix} Y_3 & Z_3 \\ Y_4 & Z_4 \end{vmatrix}+\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}\begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix} \\
-\begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}\begin{vmatrix} Y_3 & Z_3 \\ Y_4 & Z_4 \end{vmatrix}+\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}\begin{vmatrix} X_3 & Z_3 \\ X_4 & Z_4 \end{vmatrix}
\end{pmatrix}
=-\begin{pmatrix}
-\begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}\begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix}+\begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}\begin{vmatrix} X_3 & Z_3 \\ X_4 & Z_4 \end{vmatrix} \\
-\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}\begin{vmatrix} X_3 & Y_3 \\ X_4 & Y_4 \end{vmatrix}+\begin{vmatrix} X_1 & Y_1 \\ X_2 & Y_2 \end{vmatrix}\begin{vmatrix} Y_3 & Z_3 \\ Y_4 & Z_4 \end{vmatrix} \\
-\begin{vmatrix} Y_1 & Z_1 \\ Y_2 & Z_2 \end{vmatrix}\begin{vmatrix} X_3 & Z_3 \\ X_4 & Z_4 \end{vmatrix}+\begin{vmatrix} X_1 & Z_1 \\ X_2 & Z_2 \end{vmatrix}\begin{vmatrix} Y_3 & Z_3 \\ Y_4 & Z_4 \end{vmatrix}
\end{pmatrix}
$$

従って　$u-u_0$の分子と$u'-u_0$の分子　は符号だけが反対のベクトルであるから　$u-u_0$と$u'-u_0$は向きが同じかまたは正反対のベクトルである。従って３点u_0、u、u'　　が直線状に並んでいることが示された。(2016/5/7)

（例題）

$$u=\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{\begin{vmatrix}x_4-x_0 & y_4-y_0 & z_4-z_0\\x_1-x_0 & y_1-y_0 & z_1-z_0\\x_2-x_0 & y_2-y_0 & z_2-z_0\end{vmatrix}\begin{pmatrix}x_3\\y_3\\z_3\end{pmatrix}-\begin{vmatrix}x_3-x_0 & y_3-y_0 & z_3-z_0\\x_1-x_0 & y_1-y_0 & z_1-z_0\\x_2-x_0 & y_2-y_0 & z_2-z_0\end{vmatrix}\begin{pmatrix}x_4\\y_4\\z_4\end{pmatrix}}{\begin{vmatrix}x_4-x_3 & y_4-y_3 & z_4-z_3\\x_1-x_0 & y_1-y_0 & z_1-z_0\\x_2-x_0 & y_2-y_0 & z_2-z_0\end{vmatrix}}$$

$$uu=u'=\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\frac{\begin{vmatrix}x_2-x_0 & y_2-y_0 & z_2-z_0\\x_3-x_0 & y_3-y_0 & z_3-z_0\\x_4-x_0 & y_4-y_0 & z_4-z_0\end{vmatrix}\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}-\begin{vmatrix}x_1-x_0 & y_1-y_0 & z_1-z_0\\x_3-x_0 & y_3-y_0 & z_3-z_0\\x_4-x_0 & y_4-y_0 & z_4-z_0\end{vmatrix}\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}}{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\x_3-x_0 & y_3-y_0 & z_3-z_0\\x_4-x_0 & y_4-y_0 & z_4-z_0\end{vmatrix}}$$

において具体的な例で計算する。

1)　　X0＝vm,X1=v1l,X2=v1r,X3=V2l,X4=v2r のとき

$$gs1=\begin{vmatrix}x_4-x_3 & y_4-y_3 & z_4-z_3\\x_1-x_0 & y_1-y_0 & z_1-z_0\\x_2-x_0 & y_2-y_0 & z_2-z_0\end{vmatrix}=\begin{vmatrix}v2r[0]-v2l[0] & v2r[1]-v2l[1] & v2r[2]-v2l[2]\\v1l[0]-xm & v1l[1]-ym & v1l[2]-zm\\v1r[0]-xm & v1r[1]-ym & v1r[2]-zm\end{vmatrix}$$

$$gs2=\begin{vmatrix}x_4-x_0 & y_4-y_0 & z_4-z_0\\x_1-x_0 & y_1-y_0 & z_1-z_0\\x_2-x_0 & y_2-y_0 & z_2-z_0\end{vmatrix}=\begin{vmatrix}v2r[0]-xm & v2r[1]-ym & v2r[2]-zm\\v1l[0]-xm & v1l[1]-ym & v1l[2]-zm\\v1r[0]-xm & v1r[1]-ym & v1r[2]-zm\end{vmatrix}$$

$$gs3=\begin{vmatrix}x_3-x_0 & y_3-y_0 & z_3-z_0\\x_1-x_0 & y_1-y_0 & z_1-z_0\\x_2-x_0 & y_2-y_0 & z_2-z_0\end{vmatrix}==\begin{vmatrix}v2l[0]-xm & v2l[1]-ym & v2l[2]-zm\\v1l[0]-xm & v1l[1]-ym & v1l[2]-zm\\v1r[0]-xm & v1r[1]-ym & v1r[2]-zm\end{vmatrix}$$

$$u[0]=\frac{gs2*v2l[0]-gs3*v2r[0]}{gs1},u[1]=\frac{gs2*v2l[1]-gs3*v2r[1]}{gs1},u[2]=\frac{gs2*v2l[2]-gs3*v2r[2]}{gs1}$$

$$gt1=\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\x_3-x_0 & y_3-y_0 & z_3-z_0\\x_4-x_0 & y_4-y_0 & z_4-z_0\end{vmatrix}=\begin{vmatrix}v1r[0]-v1l[0] & v1r[1]-v1l[1] & v1r[2]-v1l[2]\\v2l[0]-xm & v2l[1]-ym & v2l[2]-zm\\v2r[0]-xm & v2r[1]-ym & v2r[2]-zm\end{vmatrix}$$

$$gt2=\begin{vmatrix}x_2-x_0 & y_2-y_0 & z_2-z_0\\x_3-x_0 & y_3-y_0 & z_3-z_0\\x_4-x_0 & y_4-y_0 & z_4-z_0\end{vmatrix}=\begin{vmatrix}v1r[0]-xm & v1r[1]-ym & v1r[2]-zm\\v2l[0]-xm & v2l[1]-ym & v2l[2]-zm\\v2r[0]-xm & v2r[1]-ym & v2r[2]-zm\end{vmatrix}$$

$$gt3=\begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}==\begin{vmatrix} v1l[0]-xm & v1l[1]-ym & v1l[2]-zm \\ v2l[0]-xm & v2l[1]-ym & v2l[2]-zm \\ v2r[0]-xm & v2r[1]-ym & v2r[2]-zm \end{vmatrix}$$

$$uu[0]=\frac{gt2*v1l[0]-gt3*v1r[0]}{gt1},uu[1]=\frac{gt2*v1l[1]-gt3*v1r[1]}{gt1},uu[2]=\frac{gt2*v1l[2]-gt3*v1r[2]}{gt1}$$

2)　X0＝vm,X1=v1l,X2=v1r,X3=V2lD,X4=v2l のとき

$$gs1=\begin{vmatrix} x_4-x_3 & y_4-y_3 & z_4-z_3 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}=\begin{vmatrix} 0 & 0 & v2l[2] \\ v1l[0]-xm & v1l[1]-ym & v1l[2]-zm \\ v1r[0]-xm & v1r[1]-ym & v1r[2]-zm \end{vmatrix}$$

$$gs2=\begin{vmatrix} x_4-x_0 & y_4-y_0 & z_4-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}=\begin{vmatrix} v2l[0]-xm & v2l[1]-ym & v2l[2]-zm \\ v1l[0]-xm & v1l[1]-ym & v1l[2]-zm \\ v1r[0]-xm & v1r[1]-ym & v1r[2]-zm \end{vmatrix}$$

$$gs3=\begin{vmatrix} x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}=\begin{vmatrix} v2l[0]-xm & v2l[1]-ym & -zm \\ v1l[0]-xm & v1l[1]-ym & v1l[2]-zm \\ v1r[0]-xm & v1r[1]-ym & v1r[2]-zm \end{vmatrix}$$

$$u[0]=\frac{gs2*v2l[0]-gs3*v2l[0]}{gs1},u[1]=\frac{gs2*v2l[1]-gs3*v2l[1]}{gs1},u[2]=\frac{-gs3*v2l[2]}{gs1}$$

$$gt1=\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}=\begin{vmatrix} v1r[0]-v1l[0] & v1r[1]-v1l[1] & v1r[2]-v1l[2] \\ v2l[0]-xm & v2l[1]-ym & -zm \\ v2l[0]-xm & v2l[1]-ym & v2l[2]-zm \end{vmatrix}$$

$$gt2=\begin{vmatrix} x_2-x_0 & y_2-y_0 & z_2-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}=\begin{vmatrix} v1r[0]-xm & v1r[1]-ym & v1r[2]-zm \\ v2l[0]-xm & v2l[1]-ym & -zm \\ v2l[0]-xm & v2l[1]-ym & v2l[2]-zm \end{vmatrix}$$

$$gt3=\begin{vmatrix} x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_3-x_0 & y_3-y_0 & z_3-z_0 \\ x_4-x_0 & y_4-y_0 & z_4-z_0 \end{vmatrix}=\begin{vmatrix} v1l[0]-xm & v1l[1]-ym & v1l[2]-zm \\ v2l[0]-xm & v2l[1]-ym & -zm \\ v2l[0]-xm & v2l[1]-ym & v2l[2]-zm \end{vmatrix}$$

$$uu[0]=\frac{gt2*v1l[0]-gt3*v1r[0]}{gt1},uu[1]=\frac{gt2*v1l[1]-gt3*v1r[1]}{gt1},uu[2]=\frac{gt2*v1l[2]-gt2*v1l[2]}{gt1}$$

3) X0＝vm,X1=v1l,X2=v1lD,X3=v2l,X4=v2r のとき

$$gs1 = \begin{vmatrix} x_4 - x_3 & y_4 - y_3 & z_4 - z_3 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = \begin{vmatrix} v2r[0] - v2l[0] & v2r[1] - v2l[1] & v2r[2] - v2l[2] \\ v1l[0] - xm & v1l[1] - ym & v1l[2] - zm \\ v1l[0] - xm & v1l[1] - ym & -zm \end{vmatrix}$$

$$gs2 = \begin{vmatrix} x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = \begin{vmatrix} v2r[0] - xm & v2r[1] - ym & v2r[2] - zm \\ v1l[0] - xm & v1l[1] - ym & v1l[2] - zm \\ v1l[0] - xm & v1l[1] - ym & -zm \end{vmatrix}$$

$$gs3 = \begin{vmatrix} x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \end{vmatrix} = \begin{vmatrix} v2l[0] - xm & v2l[1] - ym & v2l[2] - zm \\ v1l[0] - xm & v1l[1] - ym & v1l[2] - zm \\ v1l[0] - xm & v1l[1] - ym & -zm \end{vmatrix}$$

$$u[0] = \frac{gs2 * v2l[0] - gs3 * v2r[0]}{gs1}, u[1] = \frac{gs2 * v2l[1] - gs3 * v2r[1]}{gs1}, u[2] = \frac{gs2 * v2l[2] - gs3 * v2r[2]}{gs1}$$

$$gt1 = \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -v1l[2] \\ v2l[0] - xm & v2l[1] - ym & v2l[2] - zm \\ v2r[0] - xm & v2r[1] - ym & v2r[2] - zm \end{vmatrix}$$

$$gt2 = \begin{vmatrix} x_2 - x_0 & y_2 - y_0 & z_2 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} = \begin{vmatrix} v1l[0] - xm & v1l[1] - ym & -zm \\ v2l[0] - xm & v2l[1] - ym & v2l[2] - zm \\ v2r[0] - xm & v2r[1] - ym & v2r[2] - zm \end{vmatrix}$$

$$gt3 = \begin{vmatrix} x_1 - x_0 & y_1 - y_0 & z_1 - z_0 \\ x_3 - x_0 & y_3 - y_0 & z_3 - z_0 \\ x_4 - x_0 & y_4 - y_0 & z_4 - z_0 \end{vmatrix} = \begin{vmatrix} v1l[0] - xm & v1l[1] - ym & v1l[2] - zm \\ v2l[0] - xm & v2l[1] - ym & v2l[2] - zm \\ v2r[0] - xm & v2r[1] - ym & v2r[2] - zm \end{vmatrix}$$

$$uu[0] = \frac{gt2 * v1l[0] - gt3 * v1l[0]}{gt1}, uu[1] = \frac{gt2 * v1l[1] - gt3 * v1l[1]}{gt1}, uu[2] = \frac{gt2 * v1l[2]}{gt1}$$